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Airfoil Self-Noise Generated in a Cascade

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Self-noise from fans and rotors is generated by blade boundary-layer turbulence interacting with the trailing edges of the blades. Previous theories describing this mechanism have considered only an isolated airfoil and have shown that acoustic scattering from the sharp trailing edge is responsible for the sound that propagates to the far field. In aeroengine applications, fans or stators have relatively high solidity, and there will be acoustic scattering from adjacent blades as well as the blade locally excited by the flow. The theory is given for self-noise from a fan modeled as a linear cascade of semi-infinite flat plates. The magnitude of the self-noise mechanism is compared with the field that would be generated by a single airfoil without adjacent blade scattering.

Nomenclature

$A_p()$ = wave number transform of blade loading
a_p = pressure of excited mode
$B()$ = wave number transform of blade loading for a single blade
b = blade span
c_0 = speed of sound
$D_0 f/Dt$ = $\partial f/\partial t + U\partial f/\partial x$
d = blade overlap distance (see Fig. 1)
h = blade spacing (see Fig. 1)
$J_{\pm}()$ = split function defined as $j = J_+ J_-$
$j()$ = response function defined in Appendix A
$K_m^{\pm}$ = modal residue defined in Appendix A
$k_{0,1}()$ = response function defined in Appendix A
M = Mach number of flow
m = integer, mode order
n = blade number (see Fig. 1)
p = integer defining the incident mode orders
p_i, p_s = acoustic pressure perturbations of the incident and scattered fields, respectively
Q, q = source strength
s = blade spacing
T_{ij} = Lighthill's stress tensor
t = time
U = uniform flow velocity
y_i = position coordinates, (x, y, z)
y_0 = location of source layer
α = wave number in the y direction
β = $\sqrt{1 - M^2}$
γ = wave number in the x direction
γ_c = wave number of incident field in the x direction
Δp_n = pressure jump across each blade
ζ = $[(\omega + \gamma U)^2/c_0^2 - \gamma^2 - \nu^2]^{1/2}$
κ = $\omega/\beta^2 c_0$
κ_e = $[\kappa^2 - (\nu/\beta)^2]^{1/2}$
ν = wave number in the z direction
ρ_0 = density
ρ' = density perturbation
χ = stagger angle $\tan^{-1}(d/h)$
ω = angular frequency

I. Introduction

BROADBAND noise from fans and rotors is generated by the interaction of surfaces with turbulent flow. When the turbulent flow is generated by the blade boundary layers, the mechanism is defined as self-noise and will be the only source mechanism when the flow upstream of the fan is undisturbed. The turbulent flow generated by the blade boundary layers is convected subsonically and is an inefficient sound source in regions well away from the blade trailing edge. However, the boundary layer induces a surface pressure that is supported by the blade but cannot be supported by the wake. The sudden adjustment between these two situations generates waves that can propagate efficiently to the acoustic far field. The theory for trailing-edge noise for a single airfoil has been studied extensively[1–6] using Lighthill's acoustic analogy. Each of these theories treats the trailing-edge interaction problem in a slightly different way, but essentially they all come to the same conclusions, giving results with the same far-field directionality and flow speed dependence. It is shown that the primary noise generation mechanism is the acoustic scattering of flow disturbances at the trailing-edge discontinuity.

The theories of trailing-edge noise were originally developed for estimating the broadband noise from open rotors and aircraft wings. In these situations, the blades may be considered to be large compared with the scale of the turbulent flow. The trailing-edge interaction is well modeled by replacing the blade with a semi-infinite flat plate in an otherwise homogeneous medium. However, in aeroengine applications, fan blades cannot be considered in isolation. The blade trailing edges are often separated by distances that are smaller than the acoustic wavelength, and so there may be near-field scattering of the turbulent flow noise by more than one trailing edge. To account for this additional interaction, the fan can be modeled by a cascade of blades as shown in Fig. 1. Trailing-edge scattering of the acoustic field generated by the flow must satisfy the boundary conditions on all of the blades in the cascade, as distinct from the single-airfoil theory, where the boundary conditions need be satisfied on only one of the blades. In this paper, we will investigate the correction that must be applied to single-airfoil theories to account for scattering from a cascade of blades.

The theory developed by Amiet[3,4] for self-noise from a single blade specifies the acoustic field in terms of the convected surface pressure below the blade boundary layer. To develop a theory for self-noise generated by a cascade, we will use the same approach. However, the difference between an isolated blade and a cascade is that the former radiates waves that propagate spherically to the acoustic far field, whereas the cascade generates acoustic modes that are bounded and propagate with minimal attenuation. In the following sections, we will derive an expression for these acoustic modes for a specified convected surface pressure on one of the blades in the cascade. To illustrate the effect of the cascade, we will compare the results with the self-noise from a single bladed fan with the same diameter in which adjacent blade interference effects will not occur but the acoustic field will still be modal. The ratio of these two results will then show the importance of multiple blade interactions on self-noise from a fan.

As stated earlier, most of the theories for trailing-edge noise have modeled the blade as a semi-infinite flat plate. Amiet[3,4] also included

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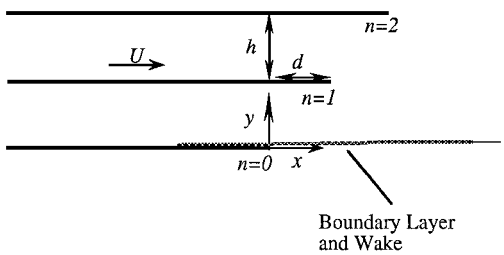

Fig. 1 Cascade model.

the effect of finite blade chord, but this is important only at low frequencies. In the cascade, waves traveling upstream are trapped in the blade passages and can propagate to the leading edge without attenuation. At the leading edge, the trapped waves will be reflected and propagate in the downstream direction, interacting with the trailing edge again and radiating away into the region downstream of the cascade. However, in this paper, we will model the blades using flat plates of semi-infinite chord so that we can identify the effects of the trailing edges in isolation. The importance of the leading-edge interaction will be considered in a subsequent publication.

To predict airfoil self-noise, the details of the flow close to the trailing edge of the blade must be known. In this region, viscous effects are important, and flow separation can occur, causing large turbulent motions of the fluid. Predicting the unsteady nature of this flowfield at high Reynolds numbers and Mach numbers, particularly at high angles of attack, is currently not possible. To address this problem, Brooks et al.[7] have developed a semiempirical method for the prediction of airfoil self-noise that is based on an extensive set of measurements of the self-noise generated by different types of isolated airfoils. Their measurements were made at a single point in the acoustic far field, and the rest of the acoustic field is inferred from the theoretical directivity patterns given by Amiet.[3,4] In the case of a cascade, the flow will scale differently from an isolated airfoil due to the proximity of the adjacent blades, and the importance of this has yet to be identified.

Acoustic waves generated by cascades of flat plates subject to an incident acoustic or vortical wave have been studied by Kaji and Okazaki,[8] Mani and Horvay,[9] Smith,[10] Koch,[11] and Peake.[12] In all of these studies, the incoming disturbance was assumed to be generated far upstream or downstream and to have no spanwise variation. Various analytical methods have been used to obtain the solution, including collocation methods[10,11] and the Wiener Hopf approach.[9,11,12] In this study, we are concerned with scattering of sound generated by a source that is next to the trailing edge of only one of the blades, and so both near-field and spanwise propagating waves will be important. The theory that is developed in Sec. II addresses both of these issues and is fundamentally different from the analyses cited earlier[8–12] because the cascade is excited by a near-field rather than a far-field source. In Sec. III, we will develop the theory for a single blade with the same source terms as were used in Sec. II so that the cascade correction factor may be derived. The most important result of this study is given by Eq. (35), which defines the ratio of the acoustic mode amplitudes for a multiple-bladed rotor relative to a single-bladed rotor. This shows the effect of the cascade response function for any type of trailing-edge flow, and the interpretation of this result is given in Sec. IV.

II. Trailing-Edge Noise from a Cascade

We will define the acoustic pressure field generated by a turbulent flow near a cascade of blades (Fig. 1) by solving Lighthill's wave equation, which is defined for stationary surfaces in a uniform mean flow as[13]

$$\frac{\tilde{D}_0^2\rho'}{\tilde{D}t^2} - c_0^2\tilde{\nabla}^2\rho' = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} - \frac{\partial}{\partial y}\sum_n \Delta p_n \delta(y - nh) \tag{1}$$

On the left-hand side of this equation, the acoustic variable is defined as the density perturbation that, in low-Mach-number flows, is directly proportional to the acoustic pressure perturbation $\rho' c_0^2$. We have also specified the wave equation in its convected form so that the quadrupole source term T_{ij} is defined using the velocity perturbations relative to a uniform flow velocity U. The terms on the left-hand side are defined using generalized derivatives so that the second source term on the right-hand side is included to account for the discontinuity in pressure between the upper and lower surfaces of each blade. We will solve this equation by considering the acoustic field to be the sum of an incident field generated by the quadrupole sources in isolation and a scattered field that is the sound generated by the pressure discontinuity. The acoustic pressure perturbation of the incident field then satisfies the equation

$$\frac{1}{c_0^2}\frac{\tilde{D}_0^2 p_i}{\tilde{D}t^2} - \tilde{\nabla}^2 p_i = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \tag{2}$$

We will assume that the quadrupole sources are confined to a boundary layer above the $n = 0$ blade and use the principle of superposition as described by Howe[5] to define the source term in Eq. (2) in the form

$$\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} = \int_{-\infty}^{\infty}\int_0^{\infty}\int_{-\infty}^{\infty} q(\gamma_c, y_0, \nu)\delta(y - y_0) \times \exp[i\gamma_c(x - U_c t) + i\nu z]\,d\gamma_c\,dy_0\,d\nu \tag{3}$$

where it is assumed that the turbulence is convected at a mean speed U_c that is slightly lower than the flow speed outside the boundary layer.

To simplify the analysis, we will limit consideration to only one harmonic component so that

$$\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} = q(\gamma_c, y_0, \nu)\delta(y - y_0)\exp(-i\omega t + i\gamma_c x + i\nu z) \tag{4}$$

with the understanding that the final results will be integrated over γ_c, y_0, and ν as in Eq. (3) and that $\omega = \gamma_c U_c$. By introducing this source term in Eq. (2) and using Fourier transforms, we obtain a solution in the form

$$p_i = Q\exp(-i\omega t + i\gamma_c x + i\zeta_c|y - y_0| + i\nu z) \tag{5}$$

where

$$\zeta_c = \sqrt{(\omega - \gamma_c U)^2 / c_0^2 - \gamma_c^2 - \nu^2}, \qquad \mathrm{Im}(\zeta_c) > 0 \tag{6}$$

and $Q = iq/(2\zeta_c)$. Because the convection velocity is subsonic, $\gamma_c > \omega/c_0$, and it follows that ζ_c is imaginary. The acoustic field generated by this disturbance therefore decays evanescently on either side of the boundary layer.

The linear cascade shown in Fig. 1 is an unwrapped model of a rotor, and so if the rotor has B blades, then the incident field will be defined by the superposition of identical source distributions periodically spaced at intervals of Bh. The incident field is then

$$p_i = Q\exp(-i\omega t + i\nu z) \times \sum_m \exp[i\gamma_c(x - mBd) + i\zeta_c|y - y_0 - mBh|] \tag{7}$$

This result shows that the incident field is periodic and so can be expressed as a Fourier series in the form

$$p_i = Q\exp[-i\omega t + i\nu z + i\gamma_c x - i\gamma_c(y - y_0)d/h] \times \sum_p F_p \exp[-2\pi i p(y - y_0)/Bh] \tag{8}$$

where the coefficients F_p are defined from Eq. (7). By considering the incident field in detail and invoking the periodicity condition, we have been able to define the incident field from a sound source in the near field as a set of harmonic waves in space and time. This is important because it allows us to use many of the concepts of traditional cascade flow analysis to solve this problem.

The scattered field is specified by the solution to the wave equation

$$\frac{1}{c_0^2}\frac{\tilde{D}_0^2 p_s}{\tilde{D}t^2} - \tilde{\nabla}^2 p_s = -\frac{\partial}{\partial y}\sum_n \Delta p_n \delta(y - nh) \tag{9}$$

subject to the boundary conditions on the blade surface and downstream of the blade trailing edges. These impose the conditions that the pressure gradient must be zero in the direction normal to the blade surfaces, and that there can be no pressure discontinuity in the blade wake. We must therefore seek a solution to Eq. (9) subject to the boundary conditions

$$\frac{\partial p_i}{\partial y} + \frac{\partial p_s}{\partial y} = 0, \qquad x < nd, \qquad y = nh$$
$$\Delta p_n(x) = 0, \qquad x > nd \tag{10}$$

To obtain a solution for the scattered field, each term of the Fourier series (8) will be treated separately so that the scattered field is expanded in the series

$$p_s = \exp(-i\omega t + i\nu z)\sum_p a_p(x, y)$$
$$\Delta p_n(x) = \exp(-i\omega t + i\nu z)\sum_p \Delta a_p^{(n)}(x) \tag{11}$$

For each term in the expansion given by Eq. (8), the incident field on each blade surface has the same amplitude but will be shifted in phase. At the same location relative to the blade trailing edge ($x - yd/h = \text{const}$), the phase shift on the blade number n relative to blade $n = 0$ will be $n\sigma = -2\pi np/B$. Consequently, it follows that the response of each blade to each incident wave component will be related by $\Delta a_p^{(n)}(x) = \Delta a_p^{(0)}(x - nd)e^{in\sigma}$. The solution to Eq. (9) is then obtained by defining the Fourier transforms

$$\tilde{a}_p(\gamma, \alpha) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_p(x, y)\exp(i\gamma x + i\alpha y)\,\mathrm{d}x\,\mathrm{d}y$$
$$A_p(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{0}\Delta a_p^{(0)}(x)\,e^{i\gamma x}\,\mathrm{d}x \tag{12}$$

so that

$$a_p(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty+i\tau}^{\infty+i\tau} \times \frac{-i\alpha A_p(\gamma)\sum_n \exp[-i\gamma(x - nd) - i\alpha(y - nh) + in\sigma]}{(\omega + \gamma U)^2/c_0^2 - \gamma^2 - \alpha^2 - \nu^2}\,\mathrm{d}\alpha\,\mathrm{d}\gamma \tag{13}$$

The integral over α can be carried out by using a closed contour in the upper or lower halves of the complex plane and applying the residue theorem. Furthermore, we can evaluate the summation analytically and write the solution in the form

$$a_p(x, y) = \int_{-\infty+i\tau}^{\infty+i\tau}\exp[-i\gamma(x - yd/h)]2\pi k^{(p)}(y, \gamma)A_p(\gamma)\,\mathrm{d}\gamma \tag{14}$$

where the function $k^{(p)}$ is defined by the summation

$$k^{(p)}(y, \gamma) = \frac{1}{4\pi}\sum_{n=-\infty}^{\infty}\operatorname{sgn}(y - nh) \times \exp[-i\gamma(y - nh)d/h + i\zeta|y - nh| + in\sigma] \tag{15}$$

which is evaluated in Appendix A. The pressure gradient in the direction normal to the surface of the blade located at $y = 0$ is then defined using

$$\frac{\partial a_p}{\partial y} = \int_{-\infty+i\tau}^{\infty+i\tau} e^{-i\gamma x}2\pi j^{(p)}(\gamma)A_p(\gamma)\,\mathrm{d}\gamma \tag{16}$$

where $j^{(p)}(\gamma) = \partial k^{(p)}(0, \gamma)/\partial y$ and is also defined in Appendix A. Taking the Fourier transform of this equation, we obtain

$$2\pi j^{(p)}(\gamma)A_p(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{0}\frac{\partial a_p}{\partial y}e^{i\gamma x}\,\mathrm{d}\gamma + \frac{1}{2\pi}\int_{0}^{\infty}\frac{\partial a_p}{\partial y}e^{i\gamma x}\,\mathrm{d}\gamma \tag{17}$$

The first integral on the right-hand side can be defined using the boundary condition upstream of the trailing edge, but the second integral is unspecified. However, the acoustic field has outgoing wave behavior, and this implies that $\partial a_p/\partial y \sim \exp(-cx)$ for large values of $x > 0$, where c is a positive constant. The second integral therefore converges[14] when $\operatorname{Im}(\gamma) > -c$. The second integral can then be defined as the function $B_+(\gamma)$ that has no singularities in the region of the complex plane where $\operatorname{Im}(\gamma) > -c$. Similarly, in the upstream region, the pressure, and hence the pressure discontinuity Δa_p, must decay as $\exp(cx)$ for large values of $x < 0$. Hence the integral that defines A_p [see Eq. (12)] will converge when $\operatorname{Im}(\gamma) < c$. Finally, by using the boundary condition to evaluate the first integral on the right-hand side of Eq. (17), we obtain

$$\frac{1}{2\pi}\int_{-\infty}^{0}\frac{\partial a_p}{\partial y}e^{i\gamma x}\,\mathrm{d}\gamma = \frac{i\xi_p QF_p\exp[i(\gamma_c d - \sigma)y_0/h]}{2\pi i(\gamma + \gamma_c)}$$
$$\operatorname{Im}(\gamma + \gamma_c) < 0, \qquad \xi_p = \left(\frac{2\pi p}{Bh} + \gamma_c d/h\right) \tag{18}$$

Consequently, there is a region of the complex plane defined by the strip $-\operatorname{Im}(\gamma_c) > c > \operatorname{Im}(\gamma) > -c$ where the functions in Eq. (17) are not singular, and this is defined as the strip of analyticity.

To obtain a solution for A_p, we will use the Wiener Hopf method, which requires that we split the function $j^{(p)}$ into two parts such that $j^{(p)} = J_+^{(p)}J_-^{(p)}$. The first part $J_+^{(p)}$ is defined so that it has no singularities or zeros when $\operatorname{Im}(\gamma) > -c$, whereas the function $J_-^{(p)}$ has no singularities or zeros when $\operatorname{Im}(\gamma) < c$. We then rearrange Eq. (17) as

$$J_-^{(p)}(\gamma)A_p(\gamma) = \frac{i\xi_p QF_p\,e^{i\xi_p y_0}}{(2\pi)^2 i(\gamma + \gamma_c)J_+^{(p)}(\gamma)} + \frac{B_+(\gamma)}{2\pi J_+^{(p)}(\gamma)} \tag{19}$$

To obtain a Wiener Hopf equation, we will separate the terms in this equation that are singular in either the upper or lower halves of the complex plane. First we note that the term on the left-hand side has no singularities in the lower half of the complex plane, whereas the last term on the right-hand side has no singularities in the upper half of the complex plane. The first term on the right-hand side has a simple pole at $\gamma = -\gamma_c$ that lies above the strip of analyticity and hence can be regarded as lying in the upper half of the complex plane. Subtracting a term from both sides of Eq. (19), which cancels the contribution from the pole at $\gamma = -\gamma_c$ on the right-hand side of Eq. (19), allows us to define a Wiener Hopf equation in which the left-hand side has no singularities in the upper half of the complex plane and the right-hand side has no singularities in the lower half of the complex plane. The right-hand side of the equation will be

$$J_-^{(p)}(\gamma)A_p(\gamma) - \frac{i\xi_p QF_p\,e^{i\xi_p y_0}}{(2\pi)^2 i(\gamma + \gamma_c)J_+^{(p)}(-\gamma_c)} \tag{20}$$

The singularities of J_-A_p must therefore be canceled by the second term in this equation. When the inverse transform of A_p is evaluated, only the singularities in the complex plane will contribute, and so the two terms in Eq. (20) can be equated giving

$$A_p(\gamma) = \frac{i\xi_p QF_p\,e^{i\xi_p y_0}}{(2\pi)^2 i(\gamma + \gamma_c)J_+^{(p)}(-\gamma_c)J_-^{(p)}(\gamma)} \tag{21}$$

We have now obtained the solution we require, and the scattered acoustic field is given by substituting Eq. (21) into Eq. (14) and summing over all terms p.

We are most interested in evaluating the acoustic field downstream of the trailing edges, and this can be obtained by evaluation of the integral in Eq. (14) using a contour closed in the lower half of the complex plane. The only singularities that contribute to these integrals are those associated with the function $k^{(p)}$, which has simple poles at the locations $\gamma = \gamma_{nB+p}^-$. The residues at these singularities are defined in Appendix A, and evaluation of the contour integral gives the scattered field as

$$p_s = \frac{-2\pi i}{h}\sum_{p,n}\exp(-i\omega t + i\nu z) \times \frac{i\xi_p QF_p K_m^-\exp\left[i\xi_p y_0 - i\gamma_m^-(x - yd/h) - 2\pi imy/Bh\right]}{(2\pi)^2 i\left(\gamma_m^- + \gamma_c\right)J_+^{(p)}(-\gamma_c)J_-^{(p)}\left(\gamma_m^-\right)} \tag{22}$$

where $m = nB + p$. This result is simplified if we note that the functions $J_{\pm}^{(p)} = J_{\pm}^{(nB+p)}$, so the summation over n can be carried out for each fixed value of m giving

$$p_s = \frac{-2\pi i}{Bh}\sum_m QH_m(\gamma_c)\exp(-i\omega t + i\nu z) \times \frac{K_m^- \exp\left[-i\gamma_m^-(x - yd/h) - 2\pi imy/Bh\right]}{(2\pi)^2 i\left(\gamma_m^- + \gamma_c\right)J_+^{(m)}(-\gamma_c)J_-^{(m)}\left(\gamma_m^-\right)} \tag{23}$$

where

$$H_m(\gamma_c) = B\sum_n i\xi_{m-nB}F_{m-nB}\exp(-i\xi_{m-nB}y_0) \tag{24}$$

Equation (23) represents the result for the scattered field that propagates downstream from the cascade. It is given in terms of a set of modes whose amplitudes are defined by each term in the series. The function H_m is given by a series summation that is slow to converge as defined by Eq. (24) and is more easily evaluated by noting that

$$H_m(\gamma_c) = B\frac{\partial}{\partial y_0}\sum_n F_{m-nB}\exp(i\xi_{m-nB}y_0) \tag{25}$$

The summation in Eq. (25) can be turned into a rapidly convergent series by using the Poisson sum formula. To demonstrate this, note that, if the function $f(y) = f(y + Bh)$ can be expanded in a Fourier series with coefficients F_p, then it follows that we can define a function

$$S_m(y) = \frac{1}{B}\sum_{k=0}^{B-1} f(y + kh)\exp(2i\pi mk/B) = \sum_n F_{m-nB}\exp[-2\pi i(m - nB)y/Bh]$$

In this case, the function $f(y)$ can be obtained from Eq. (7) and may be written in the form

$$f(y) = \exp(i\gamma_c yd/h)\sum_m \exp(-i\gamma_c mBd + i\zeta_c|y - mBh|)$$

It then follows that

$$S_m(y) = \frac{1}{B}\sum_{k=0}^{B-1}\sum_m \exp[-i\gamma_c yd/h + i\gamma_c(k - mB)d + i\zeta_c|y - (k - mB)h|]$$

This series converges rapidly because ζ_c is imaginary and approximately equal to $i\gamma_c$. In the limit that $\gamma_c h \gg 1$, only the terms $k = 0$ and $m = 0$ will be important provided that $|y_0| < h$. The function H_m can be approximated as

$$H_m(\gamma_c) = B\frac{\partial}{\partial y_0}S_m(-y_0)\exp(i\gamma_c y_0 d/h) \approx i\zeta_c\exp(i\zeta_c|y_0|) \tag{26}$$

which is independent of the mode order and greatly simplifies the evaluation of Eq. (23).

III. Single-Airfoil Scattering Response

We wish to compare the results obtained for the cascade with the trailing-edge noise from a single blade with the same boundary-layer excitation. To achieve this, we will apply the same analysis but will use only one blade so that the incident field given by Eq. (7) includes only the $m = 0$ term of the series. The boundary condition then gives

$$\frac{\partial p_s}{\partial y} = i\zeta_c Q\exp(-i\omega t + i\nu z + i\gamma_c x + i\zeta_c y_0), \qquad x < 0 \tag{27}$$

The Fourier transform of the pressure discontinuity across the blade $B(\gamma)$ is then obtained in the same way as in the preceding section with $k(y, \gamma) = \text{sgn}(y)\exp(i\zeta|y|)/4\pi$. We then obtain a result that is equivalent to Eq. (21) as

$$B(\gamma) = \frac{i\zeta_c Q}{(2\pi)^2 i(\gamma + \gamma_c)G_+(-\gamma_c)G_-(\gamma)} \tag{28}$$

where $G_+G_- = i\zeta/4\pi$ and can be defined as

$$G_+(\gamma) = (i\beta/4\pi)\sqrt{\gamma - \kappa M + \kappa_e}, \qquad G_-(\gamma) = \sqrt{\kappa_e - \gamma + \kappa M} \tag{29}$$

The scattered field can then be evaluated from the integral

$$p_s = \exp(-i\omega t + i\nu z)\frac{\text{sgn}(y)}{2} \times \int_{-\infty - i\tau}^{\infty - i\tau}\frac{i\zeta_c Q\exp(-i\gamma x + i\zeta_c y_o + i\zeta|y|)}{(2\pi)^2 i(\gamma + \gamma_c)G_+(-\gamma_c)G_-(\gamma)}\,\mathrm{d}\gamma \tag{30}$$

When a single blade is mounted in a rotor, we can specify an equivalent cascade that is the superposition of the fields from the single blade at intervals corresponding to one rotor diameter. The scattered field can then be defined as

$$p_s = \exp(-i\omega t + i\nu z) \times \int_{-\infty - i\tau}^{\infty - i\tau}\frac{i\zeta_c Q\exp[-i\gamma(x - yd/h) + i\zeta_c y_0]}{(2\pi)^2 i(\gamma + \gamma_c)G_+(-\gamma_c)G_-(\gamma)}2\pi k_0(y, \gamma)\,\mathrm{d}\gamma \tag{31}$$

where

$$k_0(y, \gamma) = \frac{1}{4\pi}\sum_{n=-\infty}^{\infty}\text{sgn}(y - nBh) \times \exp[-i\gamma(y - nBh)d/h + i\zeta|y - nBh|] \tag{32}$$

In the region downstream of the trailing edge, this integral is evaluated using a contour in the lower half-plane and has singularities only at the poles of k_0. If we compare the definition of the function k_0 with the function $k^{(p)}$ defined in Appendix A [Eq. (A3)], we see that the poles of k_0 are located at the solutions to

$$\pm\zeta h + \gamma d = 2\pi m \tag{33}$$

and so the scattered field is of the form

$$p_s = \exp(-i\omega t + i\nu z)\frac{(-2\pi i)}{Bh} \times \sum_m \frac{i\zeta_c QK_m^- \exp\left[-i\gamma_m^-(x - yd/h) - 2\pi imy/Bh + i\zeta_c y_0\right]}{(2\pi)^2 i\left(\gamma_m^- + \gamma_c\right)G_+(-\gamma_c)G_-\left(\gamma_m^-\right)} \tag{34}$$

This result shows that the field downstream of the trailing edges is purely modal. However, that is not the case in the upstream direction for which the integral in Eq. (31) is closed in the upper half-plane and must circumscribe the branch cut associated with the function G_-. The presence of the branch cut is required to satisfy the boundary conditions on the surface of the isolated blade. Interestingly, this is not the case for the full set of blades where the upstream field is purely modal. This is demonstrated by the evaluation of the integral given in Eq. (14), using a contour closed in the upper half-plane. Inspection of Eqs. (21), (A3), and (B7) shows that this is determined only by the zeros of $J_-^{(p)}$ for which the wave numbers correspond to ducted modes between the cascade blades, i.e., $\zeta = m\pi/\beta h$.

By comparing this result with Eq. (23), we see that we can define a correction factor for each mode that gives the ratio of the modal amplitude for a cascade of blades to the result obtained for a single-bladed rotor. The modal correction factor is given as

$$C_m = \left|\frac{H_m(\gamma_c)G_+(-\gamma_c)G_-\left(\gamma_m^-\right)}{\zeta_c J_+^{(m)}(-\gamma_c)J_-^{(m)}\left(\gamma_m^-\right)}\right| \tag{35}$$

This is the primary result of this paper. It shows how the trailing-edge noise from a single-bladed rotor has to be modified to allow for additional scattering by the adjacent blades. The expression is given in terms of the modal amplitudes and depends crucially on the functions $J_\pm$, which determine the cascade response.

IV. Results and Discussion

First we will compare the result obtained for a single blade in a ducted configuration, Eq. (34), with earlier results[3,5] obtained for trailing-edge noise from a semi-infinite flat plate. The result given by Eq. (30) can be shown to be identical to the result given by Amiet[3] for a convected pressure disturbance past the trailing edge of a semi-infinite flat plate. The scaling of the sound power with flow velocity and far-field directionality will therefore be identical to that given in previous studies. When this result is put into a modal form [Eq. (34)], the scaling with flow speed is more subtle. At low Mach numbers, we can take $\gamma_c \gg \gamma_m^\pm$, and for each mode specified in Eq. (34), the only terms that do not scale with the Strouhal number $\gamma_c h$ are K_m^+ and G_-. At high frequencies, K_m^+ is independent of frequency, and G_- scales as $\omega^{1/2}$. The pressure spectrum scales as U^2, and so combining these results, we expect each mode to increase in amplitude as $U^{3/2}$. To estimate the sound power output, we note that each mode will contribute independently to the sound power from the cascade, and the number of modes in the summation given by Eq. (34) will increase linearly with the frequency. However, we cannot ignore the spanwise modes that are determined by the spanwise extent of the blade and further increase the modal density such that the total number of modes increases with the square of the frequency. The modal sound power therefore will scale as U^5, as is expected for an unducted blade.[1,2,5] The presence of adjacent blades will alter these scaling arguments only by the effect of the correction factor given in Eq. (35), which will be discussed in more detail later.

Before the modal correction factor given by Eq. (35) is considered, the split functions $J_\pm$ must be evaluated, and these are given in Appendix B. The same functions are required for the calculation of the cascade response function for an incoming vortical or acoustic disturbance, and in another study, routines were developed to evaluate these functions. This code was verified by comparison with published calculations,[15,16] and exact agreement was obtained. However, the computation of these functions requires the evaluation of an infinite series that is slow to converge, but an alternate summation is possible that gives the exact result for the amplitude of these functions from a finite series of terms, and this is discussed in Appendix B.

To illustrate the behavior of the split functions, Fig. 2 gives a typical calculation of $J_+(-\gamma_c)J_-(\gamma)$ for a given frequency and Mach number. At wave numbers that lie in the range $\kappa_e + \kappa M < \gamma < \kappa_e + \kappa M$, the function is oscillatory with multiple peaks and zeros, and it is this resonant type of behavior that determines the value of Eq. (35). Outside of this range, the functions have algebraic growth, and this is seen by the trends in the curves for large and small arguments. Peake[12] has derived asymptotic expressions for the evaluation of this function, and by using his results, we can obtain a first-order approximation that is of the form

$$J_+^{(m)}(-\gamma_c)J_-^{(m)}(\gamma) = \frac{G_+(-\gamma_c)G_-(\gamma)}{[1 - \exp(i\zeta h - i\gamma d - i\sigma)]} \tag{36}$$

Using the approximation (36) then allows us to define

$$C_m \approx |1 - e^{2i\zeta_m h}| \tag{37}$$

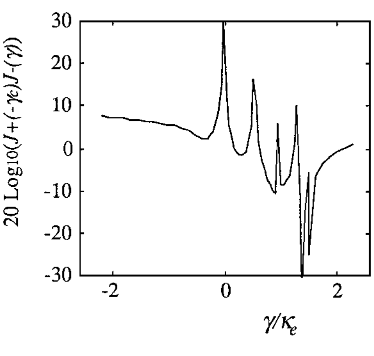

Fig. 2 Split functions as a function of wave number. Calculations are for κh = 8.15, d/h = 1.6, ν = 0, and M = 0.5.

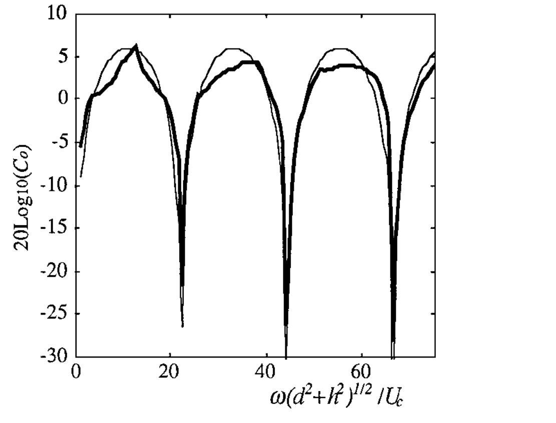

Fig. 3 Cascade correction factor as a function of nondimensional frequency based on blade spacing. Calculations are for d/h = 1.6, m = 0, ν = 0, and M = 0.5; ——, exact result, and ——, approximation by Eq. (37).

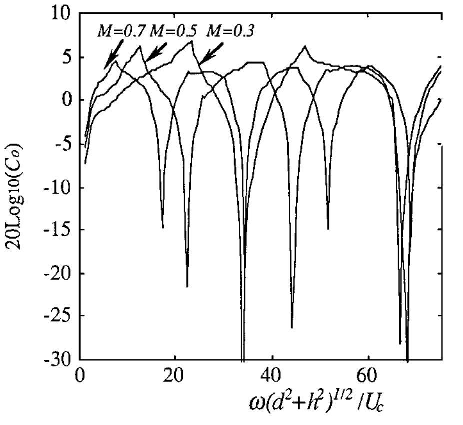

Fig. 4 Cascade correction function as a function of nondimensional frequency based on blade spacing. Calculations are for d/h = 1.6; m = 0; ν = 0; and M = 0.3, 0.5, and 0.7.

where ζ_m is the value of ζ evaluated when $\gamma = \gamma_m^-$. Equation (37) illustrates the dependence of the result on the parameters involved. It shows that C_m is oscillatory and lies in the range 0–2. The approximation is compared with exact calculations in Fig. 3 for a single mode ($m = 0$, $\nu = 0$, $M = 0.5$, and $d/h = 1.61$). First, note the nulls in these results that correspond to the zeros of Eq. (37), and second, the cascade correction gives up to 6-dB increases over and above the level from a single blade. This clearly will have an important effect on the levels of radiated noise from the cascade. The approximation given by Eq. (37) is not particularly accurate as shown in Fig. 3 but correctly identifies the more important features of the cascade response.

The approximation (37) suggests that the modal corrections are relatively independent of Mach number, and this is illustrated in Fig. 4, which shows the modal correction factor for the zero-order mode at three different Mach numbers. The nulls in these curves occur at different frequencies due to the change in κh for a fixed Strouhal number, but the maximum level of the correction factor remains the same for all Mach numbers considered. Consequently, the additional scattering by adjacent blades is not expected to alter the U^5 scaling obtained from the theory for isolated blades.

A physical explanation of the effect of adjacent blades on trailing-edge noise is illustrated in Fig. 5. On an isolated blade, the turbulent flow in the vicinity of the blade trailing edge causes an edge dipole whose axis is oriented normal to the flow direction. The presence of adjacent blades provides multiple reflecting surfaces as illustrated in Fig. 5, but to a first approximation, the effect of the cascade can be approximated by placing an infinite reflecting plane at the location of the closest neighboring blade. The reflected field is then given by an image dipole at a distance $y = 2h$ above the trailing-edge dipole, and this simplified model gives the approximate correction

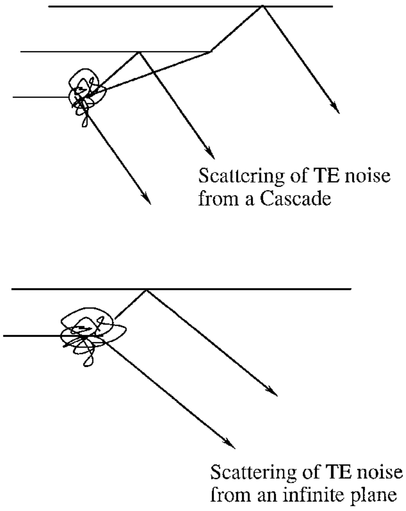

Fig. 5 First-order approximation of trailing-edge noise scattering from a cascade (top) given by the scattering by the closest blade as if it were a rigid plane (bottom).

factor (37). The presence of additional trailing edges accounts for the differences between the exact solution and the approximate solution as illustrated in Fig. 3. However, the simplified model given here explains the maximum amplification and the dips in the spectra that can be expected from adjacent blade scattering.

Finally, it is of interest to verify that the modal solution of the scattered field satisfies the Kutta condition at the trailing edge of the blades. The Kutta condition is satisfied if the flow velocity induced by each mode in the direction normal to the blade chord tends to zero just downstream of the trailing edges. To verify that this is the case, consider the linearized form of the momentum equation, which relates v, the flow velocity in the y direction, to the pressure gradient as $\rho_0 D_0 v/Dt = -\partial p_s/\partial y$. Considering Eq. (16), we find that, for each term in the series expansion of the incident field, the normal velocity can be defined as

$$v = \int_{-\infty+i\tau}^{\infty+i\tau} e^{-i\gamma x} \frac{2\pi j^{(p)}(\gamma) A_p(\gamma)}{i\rho_0(\omega + \gamma U)} \, d\gamma$$

From the theory of Fourier transforms, this velocity will tend to zero as $x^{\alpha-1}$ if the integrand is of order $\gamma^{-\alpha}$ when γ tends to infinity. Therefore, to satisfy the Kutta condition, we require that $\alpha > 1$, and this is ensured from the asymptotic forms of Eqs. (21), (A8), and (B12), which give $j^{(p)} \sim \gamma$ and $A_p \sim \gamma^{-3/2}$.

V. Conclusion

The analysis given in this paper has identified the effect of adjacent blade scattering on trailing-edge noise. The results have been compared with the radiation from a single blade. The Mach number has been found to have a relatively weak effect on the results, and so scaling with flow speed will be unaltered by the effect of adjacent blades unless there is an additional Reynolds number effect that is not considered here. The dominant mechanism for the additional scattering is shown to be caused by the presence of the closest neighboring blade acting as a hard reflecting plane, and this causes increases in levels of up to 6 dB and interference dips or nulls in the spectra.

Appendix A: Evaluation of the Response Function

In this Appendix, we will evaluate Eq. (13), which is defined as

$$a_p(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty+i\tau}^{\infty+i\tau} -i\alpha A_p(\gamma) \times \frac{\sum_n \exp[-i\gamma(x - nd) - i\alpha(y - nh) + in\sigma]}{(\omega + \gamma U)^2 \big/ c_0^2 - \gamma^2 - \alpha^2 - v^2} \, d\alpha \, d\gamma$$

where $\sigma = -2\pi p/B$. The integral over α is evaluated using the residue theorem and gives

$$a_p(x, y) = \frac{1}{2} \int_{-\infty+i\tau}^{\infty+i\tau} A_p(\gamma) \exp[-i\gamma(x - yd/h)] \sum_{n=-\infty}^{\infty} \operatorname{sgn}(y - nh) \times \exp[-i\gamma(y - nh)d/h + i\zeta|y - nh| + in\sigma] \, d\gamma \tag{A1}$$

where $\zeta = \sqrt{(\omega + \gamma U)^2/c_0^2 - \gamma^2 - v^2}$. We will choose the branch $\mathrm{Im}(\zeta) > 0$ and require ω to have a small positive imaginary part so that the summation in Eq. (A1) converges. The summation can be evaluated by using the result

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1 - z}, \qquad |z| < 1$$

so that when $0 < y < h$, we obtain

$$\sum_{n=-\infty}^{\infty} \operatorname{sgn}(y - nh) \exp[-i\gamma(y - nh)d/h + i\zeta|y - nh| + in\sigma] = \frac{\exp[-i(\gamma d - \zeta h)y/h]}{1 - \exp(i\zeta h - i\gamma d - i\sigma)} - \frac{\exp[-i(\gamma d + \zeta h)(y/h - 1) + i\sigma]}{1 - \exp(i\zeta h + i\gamma d + i\sigma)}$$

By making use of this result, we can write Eq. (13) as

$$a_p = \int_{-\infty+i\tau}^{\infty+i\tau} 2\pi k^{(p)}(y, \gamma) A_p(\gamma) \exp[-i\gamma(x - yd/h)] \, d\gamma \tag{A2}$$

where the function $k^{(p)}$ is defined as

$$k^{(p)}(y, \gamma) = \frac{1}{4\pi} \left\{ \frac{\exp[-i(\gamma d - \zeta h)y/h]}{1 - \exp(i\zeta h - i\gamma d - i\sigma)} \right\} - \frac{1}{4\pi} \left\{ \frac{\exp[-i(\gamma d + \zeta h)(y/h - 1) + i\sigma]}{1 - \exp(i\zeta h + i\gamma d + i\sigma)} \right\} \tag{A3}$$

This function has poles in the complex plane that occur at the solutions to

$$\pm\zeta h + \gamma d + \sigma = 2\pi n$$

and because the interblade phase angle is $-2\pi p/B$, these can be defined as

$$\gamma_{nB+p}^{\pm} = -A \pm \sqrt{A^2 - C^2}$$

$$A = \frac{-2\pi(p + nB)d/Bh - \omega h M/c_0}{h(1 + d^2/h^2 - M^2)} \tag{A4}$$

$$C^2 = \frac{[2\pi(p + nB)/Bh]^2 + v^2 - (\omega/c_0)^2}{(1 + d^2/h^2 - M^2)}$$

where the superscript refers to solutions in the upper or lower halves of the complex plane, respectively. The residues of $2\pi k^{(p)}$ at these poles are defined as $K_{nB+p}^{\pm} \exp[-2\pi i(nB + p)y/Bh]/h$ in which

$$K_m^{\pm} = \frac{\pm 1}{2i} \left\{ \frac{h}{[\partial(\zeta h \pm \gamma d)/\partial\gamma]|_{\gamma=\gamma_m^{\pm}}} \right\} = \frac{-\big(\gamma_m^{\pm} d - 2\pi m/B\big)/2i}{\beta^2 h\big(\kappa M - \gamma_m^{\pm}\big) - (d/h)\big(\gamma_m^{\pm} d - 2\pi m/B\big)} \tag{A5}$$

where

$$\beta^2 = 1 - M^2, \qquad \kappa = \omega \big/ \beta^2 c_0$$

Note that both $K_p^{\pm}$ and $\gamma_p^{\pm}$ are functions of Bh and d/h will be unaltered if the blade number is changed for a given rotor diameter.

By differentiating Eq. (A2), we obtain

$$\frac{\partial a_p}{\partial y} = \int_{-\infty+i\tau}^{\infty+i\tau} 2\pi k_1^{(p)}(y,\gamma) A_p(\gamma) \exp[-i\gamma(x - yd/h)]\, \mathrm{d}\gamma \tag{A6}$$

where

$$k_1^{(p)}(y,\gamma) = \frac{1}{4\pi}\left\{\frac{i\zeta \exp[-i(\gamma d - \zeta h)y/h]}{1-\exp(i\zeta h - i\gamma d - i\sigma)}\right\} + \frac{1}{4\pi}\left\{\frac{i\zeta \exp[-i(\gamma d + \zeta h)(y/h - 1) + i\sigma]}{1-\exp(i\zeta h + i\gamma d + i\sigma)}\right\} \tag{A7}$$

and for the particular case when $y = 0$, we can define $j(\gamma) = k_1^{(p)}(0,\gamma)$, where

$$j^{(p)}(\gamma) = \frac{i\zeta}{4\pi} \times \left\{\frac{(1 - e^{2i\zeta h})}{[1-\exp(i\zeta h - i\gamma d - i\sigma)][1-\exp(i\zeta h + i\gamma d + i\sigma)]}\right\} \tag{A8}$$

Appendix B: Split Functions

The response of a linear cascade of blades to an upwash gust is discussed by Koch[11] and Peake[12] and depends on the split functions $J_\pm^{(p)}$, which are defined as the Wiener Hopf factorization of the function $j^{(p)}$ given in Eq. (A8). This can be rewritten as

$$j^{(p)}(\gamma) = \frac{\zeta}{4\pi}\left\{\frac{\sin(\zeta h)}{\cos(\zeta h) - \cos(\xi d + \rho)}\right\}, \qquad \zeta = \beta\sqrt{\kappa_e^2 - \xi^2} \tag{B1}$$

where

$$M = U/c_0, \qquad \beta^2 = 1 - M^2, \qquad \kappa = \omega/c_0\beta^2$$
$$\kappa_e^2 = \kappa^2 - (\nu/\beta)^2, \qquad \xi = \gamma - \kappa M \tag{B2}$$
$$\rho = \sigma + \kappa M d, \qquad \sigma = -2\pi p/B$$

To obtain the split functions $J_\pm^{(p)}(\gamma)$, we first consider the function $\zeta \sin\zeta h$, which can be expanded as an infinite product. The expansion is

$$\zeta \sin\zeta h = \kappa_e\beta \sin(\kappa_e h\beta) \prod_{m=0}^{\infty}\left(1 - \frac{\xi}{\theta_m}\right)\left(1 - \frac{\xi}{\vartheta_m}\right) \tag{B3}$$

where

$$\theta_m = -\sqrt{\kappa_e^2 - (m\pi/\beta h)^2}, \qquad \vartheta_m = \sqrt{\kappa_e^2 - (m\pi/\beta h)^2} \tag{B4}$$

are the zeros of the function.

The denominator of $j^{(p)}$ may be expanded in a similar fashion giving

$$\cos(\zeta h) - \cos(\xi d + \rho) = [\cos(\kappa_e\beta h) - \cos(\rho)] \times \prod_{m=-\infty}^{\infty}\left(1 - \frac{\xi}{\eta_m^+}\right)\left(1 - \frac{\xi}{\eta_m^-}\right) e^{C\xi} \tag{B5}$$

where C is a constant that can be shown to be zero and $\eta_m^\pm$ are the zeros of the function defined as

$$\eta_m^\pm = -f_m \sin\chi_e \pm \cos\chi_e\sqrt{\kappa_e^2 - f_m^2}, \qquad f_m = \frac{\sigma + \kappa M d - 2\pi m}{\sqrt{d^2 + (\beta h)^2}} \tag{B6}$$

and $\tan\chi_\varepsilon = d/h\beta$.

These expansions clearly define the zeros and poles of the function $j^{(p)}$, and so to obtain the factorization, we can separate out two sets of functions that only have zeros or poles in either the upper or the lower halves of the complex γ plane. This gives the factorization as

$$J_+(\gamma) = \frac{\kappa_e\beta\sin(\kappa_e h\beta)}{4\pi[\cos(\kappa_e h\beta) - \cos(\rho)]}\frac{\prod_{m=0}^{\infty}(1 - \xi/\theta_m)}{\prod_{m=-\infty}^{\infty}\left(1 - \xi/\eta_m^-\right)} e^{\Phi}$$
$$J_-(\gamma) = \frac{\prod_{m=0}^{\infty}(1 - \xi/\vartheta_m)}{\prod_{m=-\infty}^{\infty}\left(1 - \xi/\eta_m^+\right)} e^{-\Phi} \tag{B7}$$

The function Φ must be chosen so that both $J_+^{(p)}$ and $J_-^{(p)}$ have algebraic growth as ξ tends to infinity and is given by

$$\Phi = (-i\xi/\pi)[h\beta\log(2\cos\chi_e) + \chi_e d] \tag{B8}$$

The expressions given by Eq. (B7) include infinite series that are slow to converge and so can cause numerical inaccuracies. However, an accurate form of the split function can be obtained by only considering the magnitude of these functions for real values of γ. To demonstrate this, we rearrange Eq. (B3) to define the terms of the series for which the coefficients θ_m and ϑ_m are imaginary:

$$\prod_{m=M}^{\infty}\left(1 - \frac{\xi}{\theta_m}\right)\left(1 - \frac{\xi}{\vartheta_m}\right) = \frac{\zeta\sin\zeta h}{\kappa_e\beta\sin(\kappa_e h\beta)\prod_{m=0}^{M-1}\left[1 - (\xi/\theta_m)\right]\left[1 - (\xi/\vartheta_m)\right]} \tag{B9}$$

where M is chosen so that $M\pi > \kappa_e h\beta$. Because $\theta_m = \vartheta_m^*$ for all of the terms on the left-hand side, and we have defined ξ as real, we find that

$$\prod_{m=M}^{\infty}\left|\left(1 - \frac{\xi}{\theta_m}\right)\right|^2 = \prod_{m=M}^{\infty}\left|\left(1 - \frac{\xi}{\vartheta_m}\right)\right|^2 = \frac{\zeta\sin\zeta h}{\kappa_e\beta\sin(\kappa_e h\beta)\prod_{m=0}^{M-1}\left[1 - (\xi/\theta_m)\right]\left[1 - (\xi/\vartheta_m)\right]} \tag{B10}$$

The magnitude of the slowly converging part of the series on the top line of Eq. (B7) is therefore defined exactly by a series with a finite number of terms. Similar arguments may be applied to Eq. (B5) to give

$$\prod_{m=-\infty}^{M_1}\prod_{m=M_2}^{\infty}\left|\left(1 - \frac{\xi}{\eta_m^+}\right)\right|^2 = \prod_{m=-\infty}^{M_1}\prod_{m=M_2}^{\infty}\left|\left(1 - \frac{\xi}{\eta_m^-}\right)\right|^2 = \frac{\cos(\zeta h) - \cos(\xi d + \rho)}{[\cos(\kappa_e h\beta) - \cos(\rho)]\prod_{m=M_1+1}^{M_2-1}\left[1 - (\xi/\eta_m^+)\right]\left[1 - (\xi/\eta_m^-)\right]} \tag{B11}$$

where M_1 and M_2 are chosen so that $f_m > \kappa_e$ for $m < M_1$ and $m > M_2$. By substituting the expressions (B10) and (B11) into the definition of $J_-^{(p)}(\gamma)$, we obtain

$$\left|J_-^{(p)}(\gamma)\right| = \left|\frac{\zeta\sin\zeta h}{\kappa_e\beta\sin(\kappa_e h\beta)}\left[\frac{\cos(\kappa_e h\beta) - \cos(\rho)}{\cos(\zeta h) - \cos(\xi d + \rho)}\right] \times \prod_{m=0}^{M-1}\frac{(1 - \xi/\vartheta_m)}{(1 - \xi/\theta_m)}\prod_{m=M_1+1}^{M_2-1}\frac{\left[1 - (\xi/\eta_m^-)\right]}{\left[1 - (\xi/\eta_m^+)\right]}\right|^{\frac{1}{2}} \tag{B12}$$

and $J_+^{(p)}$ can be calculated from $|j^{(p)}(\gamma)/J_-^{(p)}(\gamma)|$. This gives an exact expression for the split functions that is convergent and requires only a finite number of terms to be evaluated. It offers an enormous improvement in computational effort compared with other procedures for calculating this function but unfortunately only gives the magnitude of the function on the real axis.

There are two problems when evaluating $|J_-^{(p)}(\gamma)|$ numerically. First, at the mode wave numbers $\gamma = \xi + \kappa M = \eta_m^- + \kappa M$, the representation given by Eq. (B12) is indeterminate, and second, singularities can occur when $\xi = \theta_m$. To avoid these problems, the singular points are moved off the real axis by giving κ_e a small imaginary part and restricting γ to the real axis. Numerical calculations have shown that multiplying κ_e by $(1 + i\varepsilon)$ with $\varepsilon \sim 10^{-3}$ gives a convergent result.

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